# Table of Contents

**LEARNING OBJECTIVES**

INTRODUCTION ................................................................................................................. 4

NATURAL TOLERANCE ................................................................................................. 4
NATURAL TOLERANCE DEFINED ............................................................................... 5
COMPARING PROCESSES .......................................................................................... 5
SMALLER IS BETTER .................................................................................................. 6
CAPABILITY STUDIES ............................................................................................... 6
INDICES ...................................................................................................................... 7
CP AND Cpk ................................................................................................................ 7

CP ..................................................................................................................................... 8

VOICE OF THE CUSTOMER/PROCESS ....................................................................... 8
EXAMPLE 1 .................................................................................................................. 9
CP FORMULA ............................................................................................................... 10
CP CALCULATION ...................................................................................................... 10
CP FORMULA ............................................................................................................... 11
CP AND YIELD ............................................................................................................ 12
CP AND SIGMA .......................................................................................................... 13
CP AND SPEC CHANGES .......................................................................................... 14
CP SUMMARY ............................................................................................................. 14

CPK .................................................................................................................................. 15

CPK DEFINED ............................................................................................................. 15
CPK CALCULATION .................................................................................................... 16
CPK AND CP ............................................................................................................... 17
CP AND CPK CHANGES ............................................................................................. 17

DISCRETE DATA .......................................................................................................... 18

SIGMA LEVELS .......................................................................................................... 20
ROLLED THROUGHPUT YIELD .................................................................................. 21

CASE STUDIES ............................................................................................................ 22

CASE STUDY: PROCESS CAPABILITY ..................................................................... 22
CASE STUDY: STABILITY ............................................................................................. 23
CASE STUDY: NORMALITY .......................................................................................... 23

APPENDIX A .................................................................................................................. 24

ESTIMATING $\sigma$ FROM CONTROL CHARTS ............................................................ 24
Learning Objectives

Upon completion of this course, student will be able to:
- Compute Cp, Cpk, Pp and Ppk values for processes using continuous data
- Use these measurements to interpret the values and relate them to a defect level
- Take relevant process information for a process using discrete data
- Calculate process assessment measurements
- Determine how well the process is meeting the customer’s requirements
- Look at a very powerful operation metric called Rolled Throughput Yield

Introduction

Process capability is the extent to which a stable process is able to meet customer requirements, or specifications. But stable doesn't imply capable. Our goal is to make processes much more capable by reducing variation. The result of this will be a higher percentage of output close to target. In this module, we'll discuss ways to assess process capability for both continuous and discrete data.

Natural Tolerance

Let's start by looking at how process capability can be assessed for continuous data. In this module, we will assume the process data follows a normal distribution. If the distribution is not normal, capability can still be assessed using the Weibull distribution. To begin, we need to understand a term called natural tolerance.
Natural Tolerance Defined

Natural tolerance is a commonly used term in industry. You can think of it as the natural spread of the process.

The natural tolerance of a process extends from three standard deviations below the mean to three standard deviations above the mean, for a total of six standard deviations. The symbol for a standard deviation is the lower case Greek letter sigma (σ). Note that the terms standard deviation and sigma are sometimes used interchangeably.

Comparing Processes

Different processes can have different natural tolerances. The size of the natural tolerance of a process depends on the amount of variation.

The standard deviation of Process B is smaller than that of Process A, so the variation is less. And because the standard deviation is smaller in Process B than in Process A, its natural tolerance is also smaller.
Smaller is Better

These three processes all have different natural tolerances. The smaller the amount of variation in the process, the smaller the natural tolerance is for that process. The smaller the natural tolerance is, the better. In the following exercises, you will determine the natural tolerance of some processes.

Capability Studies

To assess capability, process owners will often perform a capability or performance study to assess a process relative to the specifications. Capability and performance indices are often used to assess how well a process is able to meet the customer’s requirements. The most widely used indices are Cp, Cpk, and Pp, Ppk. These indices are often requested by customers from their suppliers and are a common method used to measure performance.
Indices

You may have noticed that the formulas for $C_p$ and $P_p$ are identical and the formulas for $C_{pk}$ and $P_{pk}$ are also identical. This is not a typo. The difference between the indices is how we estimate sigma, the process standard deviation. $C_p$ and $C_{pk}$ are often referred to as short-term capability because of the method used to estimate the standard deviation. $P_p$ and $P_{pk}$ are often referred to as long-term performance.

From this point on we will discuss and work with $C_p$ and $C_{pk}$.

### Capability Indices

\[ C_p = \frac{USL - LSL}{6\sigma} \]

\[ C_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} \]

### Performance Indices

\[ P_p = \frac{USL - LSL}{6\sigma_{LT}} \]

\[ P_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma_{LT}}, \frac{\mu - LSL}{3\sigma_{LT}} \right\} \]

### Cp and Cpk

In the following examples, we will learn how to calculate $C_p$ and $C_{pk}$ given an estimate of sigma.
Cp

Voice of the Customer/Process

Cp compares the voice of the customer, which is represented by the spec width, with a voice of the process, which is represented by the natural tolerance. Before we get into the formula for Cp let’s look at a practical example.
Example 1

Imagine that you have to drive this big car into this little garage. The sides of the garage door are like spec limits, and the width of this car is the natural tolerance. As you can see, it won't fit.
Here's the same garage, but a much smaller car. Again, the car represents a process and the door represents the specs. In this case, the car can fit in the garage. Cp compares the variation of the process to the spec limits, like comparing the width of this car to the width of the garage door. It tells us whether the natural tolerance can fit within the spec limits.
Cp Formula

To calculate Cp, we divide the width of the specs by the natural tolerance. So Cp is equal to the upper spec limit minus the lower spec limit divided by six times one sigma. We know the spec limits, but we don't know sigma. Sigma will have to be estimated. To see various ways of estimating standard deviation (sigma), refer to the Student Guide for this module (*See Appendix A).

Cp Calculation

Now let's look at an example of a Cp calculation. We'll use the pin height data from the board pinning process. The first step is to see if the process is stable.
Cp Formula

To complete the calculation of Cp, we need to estimate sigma. Our estimate is 10. The natural tolerance of this process is six times ten, or 60. 60 divided by 60 equals one, so Cp equals one. As you can see, the width of the specs equals the natural tolerance for this process.
Cp and Yield

Now, let's see how Cp relates to yield and the level of nonconformance for centered processes. The red area in this diagram shows the part of the population which is outside the specification limits. When variation increases, Cp gets smaller and more measurements are out of spec. As variation decreases, Cp gets larger and fewer measurements are out of spec. With less variation, a larger proportion of the population is closer to target and the process is more capable.

The red area in this diagram shows the part of the population which is outside the specification limits. When variation increases, Cp gets smaller and more measurements are out of spec. As variation decreases, Cp gets larger and less measurements are out of spec. With less variation, a larger proportion of the population is closer to target and the process is more capable.
Cp and Sigma

Here is a process with spec limits at 150 and 210. Let's summarize what we just saw. When the value of sigma is 15, Cp is about 0.67, which equates to 45,500 parts per million defective. If sigma is reduced to a value of 10, Cp becomes 1.0 and the PPM value becomes 2,700. As dispersion is reduced further, sigma gets smaller, the capability index gets larger, and the PPM value gets even smaller. For centered processes, the larger Cp, the more capable the process.
Cp and Spec Changes

Notice that the spec limits are also part of the Cp formula. We have seen how Cp changes when dispersion changes. But what about when specs change. If customer requirements change, the specs may change. When specs change, Cp also changes. Take a moment to investigate the effect this specification change has on this process.

Cp Summary

The capability index Cp can be thought of as the voice of the customer divided by the voice of the process. The voice of the customer is the spec width, because specs define what the customer requires. The voice of the process is represented by the natural tolerance. Natural tolerance is a measure of variation. The spec width equals the upper spec limit minus the lower spec limit. The natural tolerance equals six times sigma. So Cp describes the potential for the process to fit in specs.
Cpk

These three processes have the same specs, the same spread, and therefore the same Cp. However, the two that are off target have more out of spec than the process that's centered on target, even though the Cp values are the same.

Cpk Defined

Cp doesn't tell us everything we need to know. We need an index that will tell us something about location. That's why we need the capability index Cpk. Cpk takes into account both dispersion and how far the process mean is from the target.

For a process that is centered on target like this one, Cpk equals Cp. Both change when variation changes. But if variation doesn't change and the process shifts, Cp stays the same but Cpk changes as the amount out of spec changes. We've already seen how to calculate Cp, now let's look at Cpk. As with Cp, Cpk requires a stable process and a normal distribution. Cpk equals the distance between the mean and the closer spec divided by half of the natural tolerance.

Let's look more closely at the Cpk formula.
Natural tolerance is 6 times sigma so half of the natural tolerance is three times sigma. The numerator is the smaller of these two distances: the mean to the lower spec limit or the upper spec limit to the mean.

Mu is the mean of a population. So, the numerator of the Cpk formula is whichever is smaller, the upper spec limit minus mu or mu minus the lower spec limit. To calculate Cpk we need the specs, mu, and sigma. The specs, we know; mu and sigma, we’ll have to estimate. Here is an example of a Cpk calculation.

**Cpk Calculation**

Let's start with the numerator. Our estimate of mu is 80. Since the process center is above target, the mean is closer to the upper spec limit. The upper spec limit is 100. 100 minus 80 is 20.

Now let's look at the denominator. We estimate sigma the same way as we would for Cp. Our estimate is 10. Three times ten is 30. So, to get the value of Cpk, divide 20 by 30. That means Cpk is about 0.67. Since this process is off center, Cpk would not equal Cp. The natural tolerance and the width of the specs are both 60, so Cp is 1.
Cpk and Cp

The formulas for Cpk and Cp are somewhat similar. The denominators both include sigma and the numerators both include spec limits. The important difference is that Cpk uses mu, the process mean. In other words, Cpk considers location of the process.

Cp and Cpk Changes

Let's look at what will happen to Cp and Cpk under some different conditions. First, let's see what happens when the dispersion changes. Notice there is a sigma in each formula. That means that when dispersion changes, both Cp and Cpk are affected. Next, let's see what happens when the process moves off target. Since mu is not in the formula for Cp, Cp won't change. But Cpk will because mu is present here. And finally, let's look at what happens when specs get tighter. Notice that specs are a part of both formulas. As specs change Cp and Cpk both change.
**Discrete Data**

Up to this point, we’ve been talking about assessing the capability of a process using continuous data. But what if your process works mostly with discrete data? How do you assess process capability for discrete data?

With discrete data, process capability is assessed using measurements such as percent defective, defects per unit, and defects per million opportunities, commonly referred to as a DPMO value.

To explain these measurements, let’s look at an example.
In this example, the unit of work is defined to be a panel with 50 holes. By the specifications, each hole must be filled with a green bead. Any color other than green is a defect. One or more defects make the panel defective.

Let’s start by calculating the percent defective panels. On these five panels, two panels contain defects. Two of the five panels are defective. That means 40% of the panels are defective.

To calculate the average number of defects per unit we just count the total number defects on all the panels and divide by the number of panels. There are seven defects on these five panels. That means the defects per unit is 1.4.

To calculate the DPMO measurement, we need to identify the number of defects and the number of opportunities. We already know there are seven defects, so we need to calculate the opportunities for error. Since each of these five panels has 50 holes, there are a total of 250 opportunities for error. Therefore, the DPMO value is 7 divided by 250 multiplied by one million. This yields a DPMO value of twenty-eight thousand.
Sigma Levels

We have seen how to express the capability of a process using both discrete and continuous data. With discrete data, we expressed the capability as percent defective, defects per unit, or a DPMO level. With continuous data, we learned how to use capability indices to express the capability and a normal curve to estimate the percent defective or PPM value. Many companies like to express the capability of both types of data as a Sigma Quality Level by relating the PPM or DPMO levels to a Sigma Level.

This chart shows the equivalent quality level or Sigma Level expressed as a number from One to Six on the horizontal axis. The defect level is shown on the vertical scale. The curved line relates the equivalent sigma level to a defect level. This defect level is often expressed as a DPMO or PPM value. Larger Sigma Quality Levels equate to more capable processes.
The chart shown here lists some processes you might be familiar with. Examine the effect on the service level of each process as the quality level improves from a 3 sigma level of quality to 6 sigma. Organizations need very capable processes when they have processes that perform millions of transactions per given time period or they have complex processes with many steps.

<table>
<thead>
<tr>
<th>Process</th>
<th>3 Sigma</th>
<th>6 Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surgical Procedures</td>
<td>33,400 incorrect surgical procedures per week</td>
<td>2 incorrect surgical procedures per week</td>
</tr>
<tr>
<td>Prescriptions</td>
<td>1,336,000 wrong prescriptions per year</td>
<td>68 wrong prescriptions per year</td>
</tr>
<tr>
<td>Airplane Landings</td>
<td>13 short or long landings at most major airports each day</td>
<td>1 short or long landing at most major airports each day</td>
</tr>
</tbody>
</table>

Examine the effect of the service level of each process as the quality level improves from a 3 sigma level of quality to 6 sigma. If you were the customer of any of these processes, which level of quality would you demand?

Rolled Throughput Yield

The previous chart showed why a 3 sigma process is not good enough for high volume processes. A 3 sigma quality level may not be good enough for complex processes either. To see why, let's look at a process that has 20 process steps or operations. For illustration purposes, we'll assume each process has the same process yield of 99%.

Some items will move through all the process steps without incurring a defect. Others will incur one or more defects and be either repaired or scrapped. As the product completes the final process step, we identify the items that are defect free and the ones that incurred a defect somewhere in the process. The object is to estimate the proportion of product that can be produced without incurring any defects. This proportion is often referred to as Rolled Throughput Yield.
To calculate Rolled Throughput Yield, we simply calculate the product of all the process step yields. In this case, we multiply .99 by itself 20 times. The result is .81791. We can now say that approximately 81.8% of the product produced by this process will be defect free throughout the process.

This chart shows the Rolled Throughput Yield for processes that contain 40 and 100 process steps. Examine the impact on Rolled Throughput Yield for more complex processes that have 99 percent yield at each process step. Also shown are the results for the same processes if the capability of each process step was improved to a three sigma or a six sigma level of quality. At Six Sigma, even a process with 100 process steps has a Rolled Throughput Yield of 99.96%.

### Case Studies

**Case Study: Process Capability**

To complete this process capability case study, you will be asked several questions based upon information available to you or from analysis you can perform using the data supplied. If you want to perform your own analysis using the raw data, click on the data button to download an Excel spreadsheet. Otherwise, you can use the information provided in the case study.
Case Study: Stability

First, we need to verify some basic assumptions. By analyzing the control charts from the process, we can conclude that both vendors have stable processes.

Evaluating Vendor Capability

Stability: Are the processes stable?

- It was determined from an examination of the control charts for each vendor that both processes were stable and capability should be assessed.
- The data for the charts used 120 subgroups of size 5 collected over 30 days.

Case Study: Normality

The team expert, using histograms and other statistical tests, verified the assumption of normality. Now we can begin assessing capability. Now we can begin assessing capability.
Appendix A.

Estimating $\sigma$ from control charts

A method for estimating standard deviation is to use the centerline on an R chart, $\bar{R}$. When estimating $\sigma$, it is important to have a stable process, so the estimate is reliable.

The process appears stable. If the process were not stable, you could not tell what it would do next. Remember, process capability is the extent to which a stable process can meet specs.

To calculate the estimate of $\sigma$, divide by a constant $d_2$. The value of $d_2$ is found in the “Table of Constants for $\bar{X}$ and R charts.” A box appears around the row corresponding to the subgroup size (n=5) used in plotting the control charts.

The value of $d_2$ is for n=5 is 2.326.
The value of $\bar{R}$ is .178.

The estimate of $\sigma$ is

$$\frac{\bar{R}}{d_2} = \frac{.178}{2.326} = .07653$$